The Game of Pig: Lesson in Probability, Statistics, and Computer Science

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Introduction

The game of Pig is commonly credited to a magician by the name of John Scarne in the year 1945. It is a dice game with many variations in play. The simplest variation, which will be studied in this lesson, is played as follows. In each turn, the player rolls a single die repeatedly until a 1 is rolled or the player decides it is time to "hold".

- In the event that the player rolls a 1, the player scores nothing and it is then the next player's turn.
- In the event that the player rolls any other number, the number is added to the player's "turn score", and the player continues with his turn.
- When the player decides to "hold", the player's turn total is added the player's "game total". It then becomes the next player's turn.

The first player with a game total of 100 points or more wins.

As an example, suppose that Greg begins his next turn with a game total of 25 points. Greg rolls a 6. He could "hold" and have 6 points added to his game score. But, instead, he chooses to roll again. He rolls a 1, and his turn ends without adding his turn total of 6 to his game total. On the other hand, suppose that Stephanie, the next player, has a game total of 17 points. She rolls the sequence 4/2/5/6/3/3 and decides to hold. Her turn total of 23 is then added to her game total, giving her a game total of 40.

The game of Pig is a jeopardy type of dice game, according to the game designer Reiner Knizia. Virtually everyone in the United States is familiar with the TV game show called *Jeopardy!*. In this game show, contestants jeopardize previous wins by attempting to gain potentially greater wins. This is clearly true of the game of Pig. A player can build up a nice turn score and risk losing that by "acting like a pig"—being greedy and rolling again—but getting a "1" and losing everything gained in that turn.

Decisions, decisions! What are my opponents' scores? My scores? Hold or roll again? Yikes!

OzoBlockly and the Game of Pig

The accompanying OzoBlockly program, *The Game of Pig.ozocode*, is loaded into Evo. Then Evo is placed at the start location of the Ozomap in the direction shown by the arrow, as shown in Figure 1. A full page Ozomap file, *The Game of Pig Ozomap.pdf*, suitable for working with Evo, accompanies this lesson.

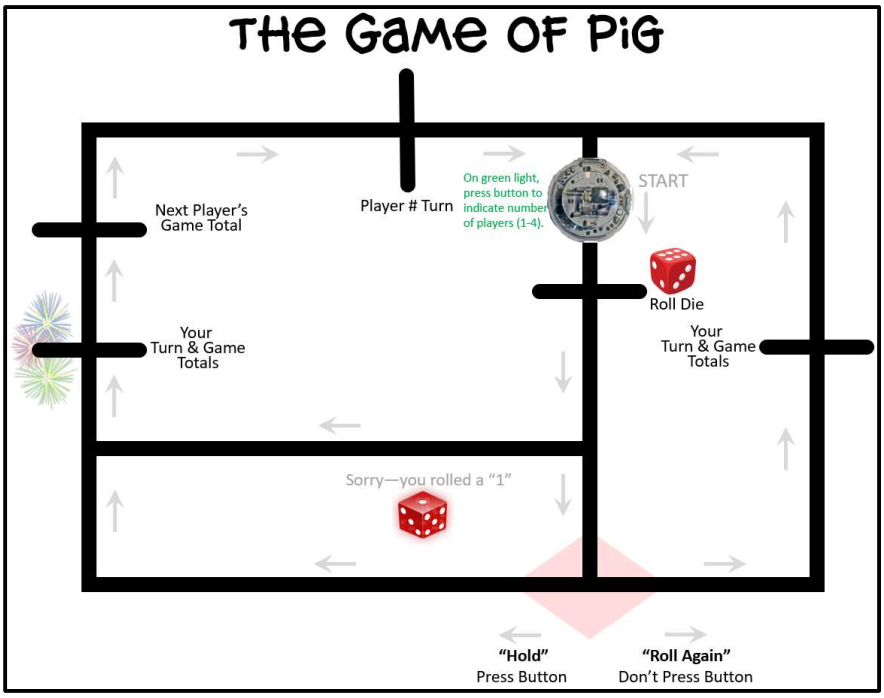

Figure 1

The program is started. When the top light turns green, the player has five seconds to indicate the number of players. Press Evo's button once for each player. The top light will quickly flash white to indicate that it recognized the button press. After the five seconds have elapsed, Evo will speak the number of players. Then he will move to the location where the die is rolled. It is now player number 1's turn. Play proceeds based upon the player's decision whether to roll again or hold. If the player intends to hold, he or she must press Evo's button once within five seconds. Otherwise, Evo will assume that the player wants to roll again. No dilly-dallying!

Evo will keep track of each player's turn and game scores and remind the player of these scores at appropriate points during game play. Evo will announce whose turn it is when encountering the "Player # Turn" intersection. The first player to meet or exceed 100 game points is the winner and will be greeted by a fireworks display!

Ozobot "Game of Pig" Just for Fun

The game of Pig with Ozobot can be played "just for fun"—no need to know OzoBlockly programming, since *The Game of Pig.ozocode* is provided with this lesson. Although as many as ten students at a time can play the game with a single Evo, it is recommended that two students compete with one another using a single Evo. Although the points required to be the winner are set to 100, this can be reduced to a smaller number if play time is an issue. For example, to change this to 50 points, adjust the block 

that is near the beginning of the program to a smaller value, say 50, for example. Discussion could be centered on strategies for winning Pig. When does knowing your competitor's score become important?

Ozobot "Game of Pig" in a Statistics and Probability Class

The game of Pig provides for some interesting studies in statistics and probability. The value obtained on a roll of the die has nothing to do with the value on the previous roll or the value on the next roll. In everyday language, we can say that these events "have nothing to do with one another". Mathematicians formally describe these events as being *independent*. They have also proven that if two or more events are independent, the probability of their simultaneous occurrence is the product of their probabilities. So—how can we apply this to the game of Pig?

Since rolling a 1 in this game means losing accumulated turn points and ending your turn, in would be interesting to investigate the following question: *What is the probability that a 1 is not rolled until roll number n, in a sequence of n rolls of a die?*

When *n* = 1, the probability is $\frac{1}{6}$.

When *n* = 2, the probability is $\frac{5}{6} \bullet \frac{1}{6} = \frac{5}{36}$, since the probability of **not** rolling a 1 on the first roll is $\frac{5}{6}$ and the probability of then rolling a 1 on the second throw is $\frac{1}{6}$. Since these events are independent, then the probability of both is the product of their probabilities.

When *n* = 3, by similar reasoning, the probability is $\left(\frac{5}{6}\right)^2 \bullet \frac{1}{6} = \frac{25}{216}$.

From the pattern suggested by the above cases, it can reasonably be concluded that the probability that a 1 is not rolled until roll number *n* in a sequence of *n* rolls of a die is:

$$\left(\frac{5}{6}\right)^{n-1} \bullet \frac{1}{6} \quad \textit{(Formula 1).}$$

A second question that would be even more of interest to a player of the game of Pig is: *What is the probability that the **first** one is rolled **on or before** roll number n in a sequence of n rolls of a die?* To answer this question, we need to get the cumulative distribution for Formula 1:

$$\sum_{k=0}^{n-1} \left(\frac{5}{6}\right)^k \left(\frac{1}{6}\right) \quad \textit{(Formula 2)}.$$

Spreadsheet Exercise

Set up a spreadsheet using your favorite spreadsheet software that will compute the individual probabilities of Formula 1 as *n* goes from 1 through 50. Then add a column to your spreadsheet that computes the cumulative probabilities as *n* goes from 1 through 50. Construct a scatter plot of probability versus *n*. Finally, answer the following questions:

1. What is the probability of rolling a 1 on roll 1?
2. What is the probability of rolling a 1 on or before roll 2?
3. What is the probability of rolling a 1 on or before roll 3?
4. On what roll number is this probability approximately 0.5?
5. On what roll number does this probability just exceed 0.8?
6. On what roll number does this probability just exceed 0.9?
7. On what roll number does this probability just exceed 0.95?
8. On what roll number does this probability just exceed 0.99?

Ozobot "Game of Pig" in a Computer Science Class

Since rolling a 1 in the game of Pig means losing accumulated turn points and ending your turn, in would be interesting to investigate the following question: *What are the chances that a 1 is not rolled until roll number n, in a sequence of n rolls of a die?*

Design an OzoBlockly program for Evo that addresses this question. Have Evo simulate rolling an ordinary six-sided die until a 1 is rolled, keeping track of the roll number on which the 1 was rolled. If the 1 occurred on roll number 4, then increase an array element with index 4 by 1. If the 1 occurred on roll number 12, then increase the array element with index 12 by 1. Repeat this process 500 times. After the 500th time, have Evo speak the values of the array starting at index 1 and proceeding through index 50.

As an indicator of Evo's progress, it is interesting to add a quick beep after each of the 500 trials. You might also want to wait for a button press to start Evo's recitation of countsnj after Evo has completed all 500 trials. Finally, it is a good idea to allow a 2-second delay between each of Evo's count recitations to provide time to jot down the counts.

Spreadsheet Exercise

Evo's counts can be jotted down on a piece of paper while Evo recites, and then they can be recorded to a column in a spreadsheet. If you add a new column by dividing each of these values by 500, you then have a spreadsheet that empirically answers the question: *What are the chances that a 1 is not rolled until roll number n, in a sequence of n rolls of a die?*

Add a final column to your spreadsheet that computes the cumulative chances, *i.e.*, a running total of the chances. This column will answer a question that would be of great interest to a Pig player: What are the chances that the first 1 is rolled ON OR BEFORE a given roll number? Construct a scatter graph of the cumulative chances versus roll number?

Optional Exercise for Students of Both Probability and Statistics and Computer Science

Plot both the theoretical probability and empirical chances that the first 1 is rolled ON OR BEFORE a given roll number ***on the same graph***. Is the agreement between the two good? It should be, if no errors were made in computing the theoretical probabilities or in designing the OzoBlockly program.