Student Investigation

Learning Math Facts through Pattern Recognition

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1. Will Evo be able to guess the cancelled digit with ***any*** pair of numbers? Try it on Evo using the numbers 7519 and 354. Was Evo correct? *No*

2. Is there something special about the numbers in the list presented on the Ozomap? What do you get if you add the digits in each of the eight numbers in the list?

   459 18 216 9 369 18 1413 9 423 9 5229 18 315 9 63 9

3. Hmm! Anything special about these *sums*? Are they all divisible by 9? Yes

4. Is it possible that the *numbers* in the list are all also divisible by 9? For example, 459/9 = 51, with a remainder of 0, so 459 is divisible by 9. What about the rest of the numbers in the list? Are they divisible by 9?

   216 Yes 369 Yes 1413 Yes 423 Yes 5229 Yes 315 Yes 63 Yes

5. What about multiples of 9 that you are familiar with: 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99, and 108? These numbers are all divisible by 9. What about the sum of the digits in each of these numbers—are the sums divisible by 9? Yes

6. Could it be possible that any number divisible by 9 also has a sum of digits that is divisible by 9 and vice versa? Consider the numbers 62,973 and 58,869? Is 62,973 divisible by 9? Yes Is 58,869 divisible by 9? Yes What is the sum of the digits in 62,973? 27 What is the sum of the digits in 58,869? 36 Are these sums divisible by 9? Yes

7. Try 62,973 and 58,869 as a number pair with Evo on the Ozomap. Was Evo correct? Yes

8. Are 9,776,385 and 26,172,369 divisible by 9? Yes Does each have a sum of digits equal to a multiple of 9? Yes Try them with Evo on the Ozomap. Was Evo Correct? Yes

9. Mathematicians have proven that any number divisible by 9 also has a sum of digits that is divisible by 9 and vice versa. In our lesson we have been *adding together two numbers each divisible by 9*. Can you prove that *their sum must also be divisible by 9*? Show your proof below.

   Let a and b each be divisible by 9.
   Then a = 9x and b = 9y, where x and y are integers.
   Then a + b = 9x + 9y = 9(x + y) = 9z, where z is an integer.
   Therefore, a + b is divisible by 9.

10. Now you are ready to figure out Evo's secret for determining the cancelled digit! Without using Evo and the Ozomap, write down the digit you cancelled and the uncancelled sum for each of the number pairs, when added together, in the table below. *[Values in the table will depend upon which digit the student cancels in the sum.]*

Number Pair	***Cancelled digit***	***Uncancelled sum***
5229 and 63	5	13
423 and 1413	8	10
18 and 9	7	2
315 and 369	4	14
62,973 and 58,869	8	10
9,867,384 and 17,295,885	3	42

What can you say about the sum of the cancelled digit and uncancelled digits for each of the number pairs in the table? *The sum is always a multiple of 9. Alternatively, the sum is always divisible by 9.*

11. You should now be able to state a rule that Evo could use to determine the cancelled digit if Evo knows the sum of the uncancelled digits. Keep in mind that you have discovered this rule through *pattern recognition*!

    To determine the cancelled digit, subtract the uncancelled sum from its next larger multiple of 9.
    For example, for 315 and 369, the cancelled digit = 18 – 14 = 4.

12. Finally, consider the pair of numbers 81 and 99. Both are divisible by 9 and both have a sum of digits equal to 9. The sum of the two numbers is 180. Try this number pair with Evo and cancel the 0 in 180. Does Evo get the cancelled digit correctly? __No__

    The reason for this is because there is ambiguity when the cancelled digit is 0 or 9. To avoid this problem, the numbers in the list should be chosen so that the digit 0 does not appear in the sum of any pair within the list. The list of numbers on the Ozomap was chosen with this in mind.