

Student Investigation

Learning Math Facts through Pattern Recognition

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1. Will Evo be able to guess the cancelled digit with **any** pair of numbers? Try it on Evo using the numbers 7519 and 354. Was Evo correct? _____
2. Is there something special about the numbers in the list presented on the Ozomap? What do you get if you add the digits in each of the eight numbers in the list?
459____ 216____ 369____ 1413____ 423____ 5229____ 315____ 63____
3. Hmm! Anything special about these *sums*? Are they all divisible by 9? _____
4. Is it possible that the *numbers* in the list are all also divisible by 9? For example, $459/9 = 51$, with a remainder of 0, so 459 is divisible by 9. What about the rest of the numbers in the list? Are they divisible by 9?
216____ 369____ 1413____ 423____ 5229____ 315____ 63____
5. What about multiples of 9 that you are familiar with: 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99, and 108? These numbers are all divisible by 9. What about the sum of the digits in each of these numbers—are the sums divisible by 9? _____
6. Could it be possible that any number divisible by 9 also has a sum of digits that is divisible by 9 and vice versa? Consider the numbers 62,973 and 58,869? Is 62,973 divisible by 9? ____ Is 58,869 divisible by 9? ____ What is the sum of the digits in 62,973? ____ What is the sum of the digits in 58,869? ____ Are these sums divisible by 9? _____
7. Try 62,973 and 58,869 as a number pair with Evo on the Ozomap. Was Evo correct? _____
8. Are 9,776,385 and 26,172,369 divisible by 9? _____ Does each have a sum of digits equal to a multiple of 9? ____ Try them with Evo on the Ozomap. Was Evo Correct? _____
9. Mathematicians have proven that any number divisible by 9 also has a sum of digits that is divisible by 9 and vice versa. In our lesson we have been *adding together two numbers each divisible by 9*. Can you prove that *their sum must also be divisible by 9*? Show your proof below.

10. Now you are ready to figure out Evo's secret for determining the cancelled digit! Without using Evo and the Ozomap, write down the digit you cancelled and the uncanceled sum for each of the number pairs, when added together, in the table below.

<i>Number Pair</i>	<i>Cancelled digit</i>	<i>Uncanceled sum</i>
5229 and 63		
423 and 1413		
18 and 9		
315 and 369		
62,973 and 58,869		
9,867,384 and 17,295,885		

What can you say about the sum of the cancelled digit and uncanceled digits for each of the number pairs in the table? _____

11. You should now be able to state a rule that Evo could use to determine the cancelled digit if Evo knows the sum of the uncanceled digits. Keep in mind that you have discovered this rule through *pattern recognition*!
12. Finally, consider the pair of numbers 81 and 99. Both are divisible by 9 and both have a sum of digits equal to 9. The sum of the two numbers is 180. Try this number pair with Evo and cancel the 0 in 180. Does Evo get the cancelled digit correctly? _____

The reason for this is because there is ambiguity when the cancelled digit is 0 or 9. To avoid this problem, the numbers in the list should be chosen so that the digit 0 does not appear in the sum of any pair within the list. The list of numbers on the Ozomap was chosen with this in mind.