Random Walk: An Experiment in Probability and Statistics

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Introduction

Games of chance provide instructive problems for anyone. Tossing a coin, rolling a pair of dice, or selecting a random card from a well-shuffled deck of playing cards are common examples. They all provide experience with fluctuations and regularities. They also show how order can emerge from what seems to be chaos. Indeed, personal experience of random processes is best accomplished by doing some simple experiments, recording results, and analyzing them.

The random walk is one of the most common as well as interesting such processes to study. Imagine that a woman stands facing East and tosses a coin to decide whether she should take one step East or one step West. She continues this process for a total of 25 steps. There are many questions we could ask regarding her random walk. Here are four such questions:

- After completing the 25-step random walk, how far would she from her starting point on average?
- On average, on how many steps is she to the East of her starting point?
- On average, on how many steps is she to the West of her starting point?
- How many times does she return to the starting point during her random walk?

Before doing an experiment, you should write down a thoughtful guess as to the outcome. This will give you experience in making estimates. Whenever your guesses are contrary to your observations from actual experiment, you should try to identify aspects of the problem that you did not take into consideration.

Flipping a coin 25 times uses up a fair amount of time and doesn't let us visualize what a random walk looks like. Here is where Evo comes into the picture. Evo has a random number generator built in, so he can simulate tossing a coin at lightning speed. Best of all, we can place him on a number line and watch him as he takes random steps East or West. Finally, you and your fellow students can collect data to answer the questions posed above while observing Evo as he moves about on the number line. See Figure 1, which shows Evo on a number line at location 3.

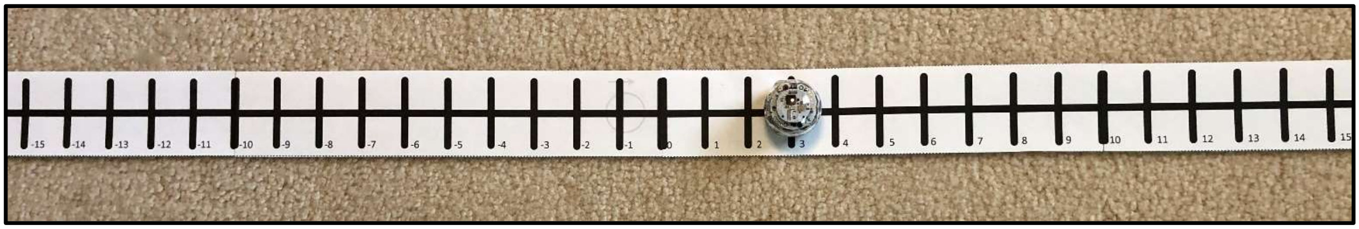

Figure 1

Your Thoughtful Guesses

Before actually doing the experiment with Evo, you should make some thoughtful guesses for answers to the questions proposed in the Introduction:

- After completing the 25-step random walk, how far would Evo be from his starting point on average? _______
- On average, on how many steps is Evo to the East of the origin? ______
- On average, on how many steps is Evo to the West of the origin? ______
- How many times does he return to the origin during his random walk? _______

Creating the Number Line and Working with the OzoBlockly Program

Figure 2 shows a reduced-size printout of the file *NumberLine.pdf*. The full-size printout is two full pages in size. To make the number line for use with this experiment: (1) cut long the dotted lines with a scissors, and (2) tape the pieces together so that they are in order from -25 to 25. You will then have a number line for Evo to traverse during this experiment. Note that there is a starting circle for Evo at location -1. Before starting the program *Random Walk.ozocode*, make sure that Evo is on this circle and facing the direction of the arrow.

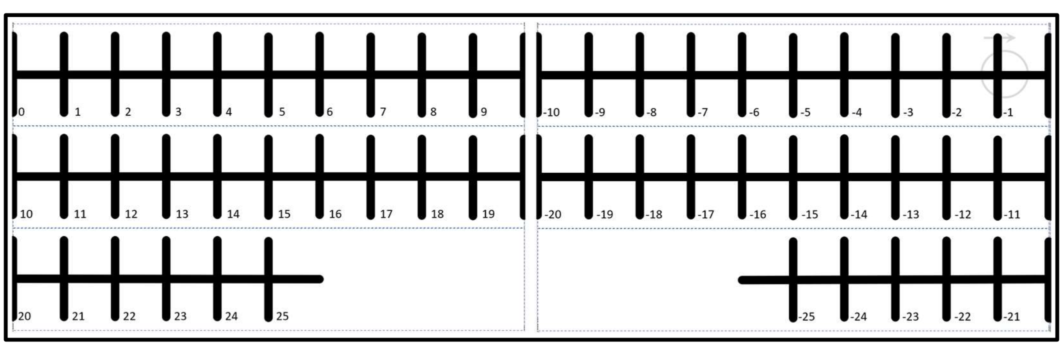

Figure 2

After the program is started, Evo will move to the origin at location 0 and make a short high-pitched beep. This indicates the start of the walk. Thereafter, with each step of the walk, Evo will make a short low-pitched beep. The default number of steps is 25, so Evo will make a low pitch beep a total of 25 times, and then stop and turn off automatically.

Data Collection

During Evo's 25-step walk, your lab group will need to collect data that allows answering the four questions posed earlier. It is suggested that one lap partner count how many steps Evo is to the East of the origin? Another lab partner can count how many steps Evo is to the West of the origin? A third lab partner can count the number of times that Evo returns to the origin. Your group will collect this data for ten random walks of 25 steps each, and then compute averages for these ten walks. Use the data table of Figure 3 to record your data. *As a check on the counts made by your lab group, note that the sum of the times East, times West, and times returning to the origin* ***for each walk*** *should be 25*. It is easy to do the counts if you count when you hear the low-pitched beep from Evo. The final distance from the origin is always taken as a positive number. For example, if the final location is -7, then the distance from the origin is 7.

Random Walk	Final Distance from Origin	Times East of the Origin	Times West of the Origin	Times Returning to the Origin
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
Totals →				
Averages →				

Figure 3

Data Analysis and Discussion

- Mathematicians tell us that the theoretical average distance from the origin after a walk of *n* steps, for large *n*, is about $0.8\sqrt{n}$.
  - How does your average compare to the theoretical value of $0.8\sqrt{25}$ = 0.8•25 =4?
  - How does your average compare to your prediction before doing the experiment?

- Now consider the averages for the times East of the origin and times West of the origin.
  - How well do these averages compare to your predictions?
  - Are these two averages reasonably close to the same value? If not, can you come up with any possible reasons for the difference?
  - For any given walk, do you notice in most cases, that a fairly substantial fraction of the time is likely to be spent on one side of the origin? Mathematicians tell us that this feature, surprising as it is, is inherent to this random walk problem.

- Finally, let's consider times at the origin.
  - How well does the average times at the origin compare to your predictions?
  - Mathematical theory tells us that for a 25-step walk, the average number of times at the origin is approximately 3.

Additional Investigation *(≈ 40 minutes)*

Time permitting, you might find it interesting to further investigate the rule that the theoretical average distance from the origin after a walk of *n* steps, for large *n*, is about $0.8\sqrt{n}$.

You can adjust the number of steps in the walk taken by Evo in the OzoBlockly block near the beginning of the program that looks like this: `set numberOfSteps to 25`. Do an experiment with 50 and 100 step walks, recording your results in the table of Figure 4.

Distance from the Origin											
Walk→ Steps↓	1	2	3	4	5	6	7	8	9	10	Average
50											
100											

How well do the averages compare to the formula $0.8\sqrt{n}$?