Quantum Tunneling Teacher Guide

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The file *Tunneling Simulation.xlsx* is a skeleton file in which data from individual groups can be entered for quick data analysis. The file appears as in Figure 1 when it is opened. Student data on width and tunneling probability are to be recorded into columns A and B, data on mass and tunneling probability in columns N and O. Graphs will automatically be filled in with exponential trend lines.

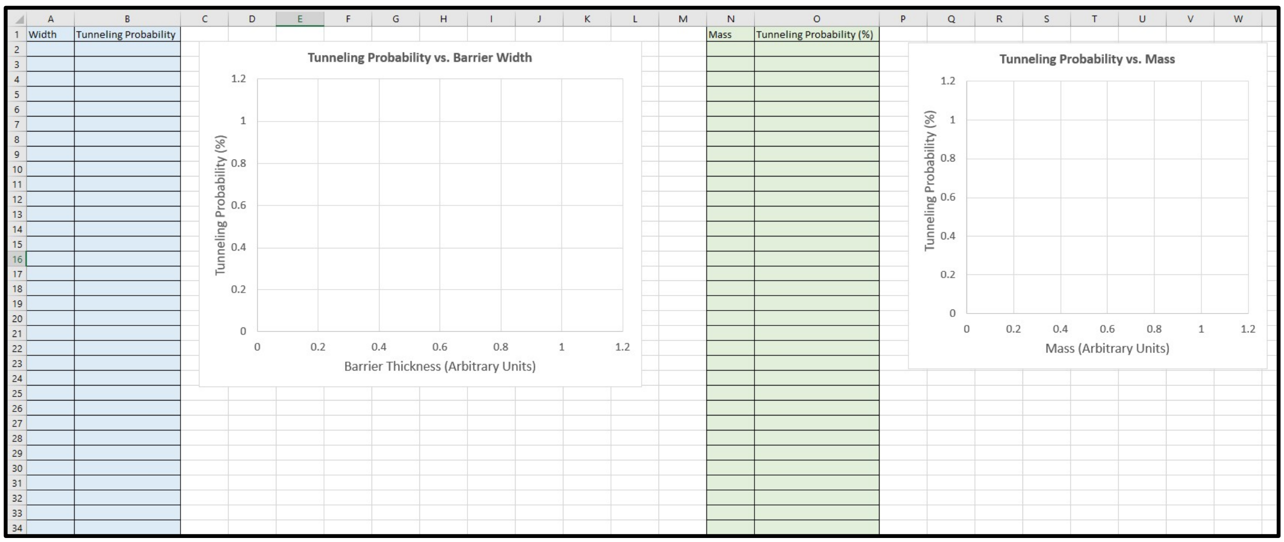
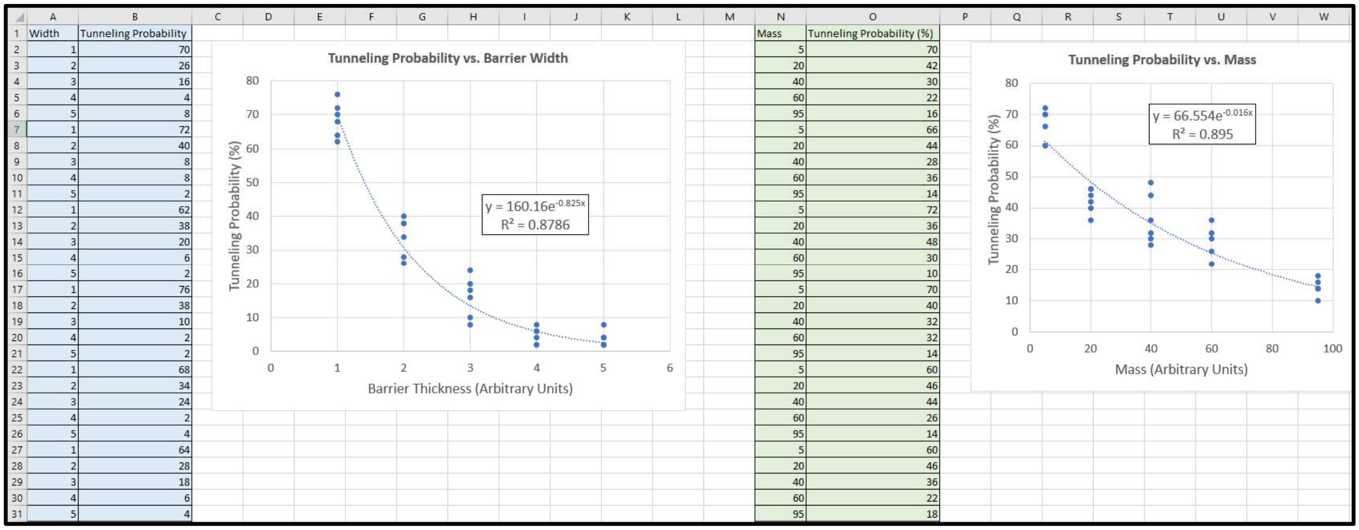


Figure 1

Figure 2 shows what the spreadsheet looks like after data from six lab groups have been recorded. The graphs will automatically update in real-time as the student data is entered. The graphs show the random variability of the data for any given value of the independent variable. More importantly, the graphs show a trend as the value of the independent variables increase. Exponential trendline fits to the data are good, with R^2 values close to 0.9.

We conclude that tunneling probability decreases exponentially as barrier width and particle mass increase.



Width	Tunneling Probability	Mass	Tunneling Probability (%)
1	70	5	70
2	26	20	42
3	16	40	30
4	4	60	22
5	8	95	16
1	72	5	66
2	40	20	44
3	8	40	28
4	8	60	36
5	2	95	14
1	62	5	72
2	38	20	36
3	20	40	48
4	6	60	30
5	2	95	10
1	76	5	70
2	38	20	40
3	10	40	32
4	2	60	32
5	2	95	14
1	68	5	60
2	34	20	46
3	24	40	44
4	2	60	26
5	4	95	14
1	64	5	60
2	28	20	46
3	18	40	36
4	6	60	22
5	4	95	18

Figure 2

Answers to Questions

Question 1 - What do you conclude about the mathematical nature of the relationships between tunneling probability and the independent variables barrier width and particle mass?

[Tunneling probability decreases exponentially as barrier width and particle mass increase.]

Question 2 - Suppose that an electron and a proton with the same kinetic energy encounter identical potential barriers. Which one will tunnel through the barrier more easily? Why?

[The mass m_e of an electron is 9.11×10^{-31} kg. The mass m_p of a proton is 1.67×10^{-27} kg. The ratio mp/me is about 1800. Since the electron is about 1800 time less massive than the proton, we would expect the electron to have a higher tunneling probability.]

Question 3 - The probability that an object will tunnel through a barrier is given approximately by the equation

$$P = e^{-2KL},$$

where *L* is the width of the barrier and *K* is the wave number. The wave number is given by the equation

$$K = \frac{2\pi\sqrt{2m(V-E)}}{h},$$

where *m* is the mass of the object, *V* is the potential energy of the barrier, *E* is the kinetic energy of the tunneling object, and *h* is Plank's constant.

Suppose that you have a mass of 60 kg, are moving at 1 m/s toward a barrier of width 1 m, and are trying to tunnel through a 40 Joule potential barrier. (A 40 Joule potential barrier is equivalent to tossing a 1 kg object 4 meters high, from the familiar equation $V = mgh$.) What is the probability that you tunnel through?

Your mass is 60 kg. Your kinetic energy $E = \frac{1}{2}mv^2 = 30$ *Joule. The potential energy of the barrier is 40 Joule. The value of Planck's constant* $h = 6.63 \times 10^{-34}$ *Joule-s. Therefore,* $K = 3.28 \times 10^{35}\ m^{-1}$. *Therefore,* $P = exp(-2KL) = exp(-3.28 \times 10^{35})$. *This is an **extremely small probability**.*

Question 4 - Consider an electron having a kinetic energy E of 4.75 eV. It is approaching a barrier with potential *V* of 5.00 eV and a width *L* of 1000 pm (picometer = 10^{-12} m). Since $E<V$, classical physics tells us that the probability is zero that it passes through the barrier. Calculate the quantum probability of the electron tunneling through the barrier.

1 eV = 1.6×10^{-19} Joule. $m_e = 9.11 \times 10^{-31}$ kg. Therefore, $K = 2.56 \times 10^9\ m^{-1}$. 1000 pm = 10^{-9} m. Therefore, $P = exp(-2KL) = exp(-2 \times 2.56 \times 10^9 \times 10^{-9}) = exp(-5.12) = 0.006 = 0.6\%$. *In other words, about 6 out of every 1000 electrons would tunnel the barrier.*