Physics: Quantum Tunneling Simulation Lesson

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Introduction

Suppose that you are on a walk and encounter a hill. The only way you are going to get to the other side of the hill is if you have enough energy to climb over the hill. Snapping your fingers will not magically get you to the other side of the hill. Alternatively, consider giving a ball a push to get it over the hill. If you don't give it a hard enough push, it will not have enough velocity to make it over the hill. And you certainly don't expect it to suddenly teleport itself to the other side of the hill in Star Trek fashion! Why not? Because you believe in the laws of *classical mechanics*.

The rather strange laws of *quantum mechanics*, however, allow subatomic particles such as electrons and protons to break through barriers. Such particles sort of dig their way through the barrier in a process known as *tunneling*. On a quantum scale, objects exhibit *wave-particle duality*, revealing properties of both waves and particles. The wave function of an electron, for example, can extend to the other side of a barrier. This wave tells us the probability of finding the electron at a specific location, *even on the other side of the barrier*.

Lesson Goals

This lesson will let you study quantum tunneling using provided OzoBlockly programs. These two programs *simulate* quantum tunneling. Evo plays the role of a subatomic particle heading toward a barrier. With one of the two programs, the probability of tunneling a barrier is studied for barriers of varying widths. With the other program, the probability of tunneling is studied for different particle masses.

You will discover that the probability of tunneling the barrier *decreases exponentially with the thickness of the barrier and with the mass of the particle*. These probabilities are derivable from the Schrödinger Equation, named after a famous Austrian-Irish physicist who shares the title of father of quantum mechanics with Max Planck and Werner Heisenberg.

Working with the OzoBlockly Programs

The left side of Figure 1 shows one of the Ozomaps that is used with the OzoBlockly program *Tunneling Barrier Width.ozocode*. Evo (the subatomic particle) is placed at the start location facing the direction of the arrow. The program is started. When the top light turns green, you have 8 seconds to press the button the number of times equal to the barrier thickness. Thickness values are 1, 2, 3, 4, and 5. Each time you press the button, the top light will quickly flash white to let you know that Evo recognized the button press. After the 8 seconds has elapsed, Evo will speak the value of the barrier width and begin moving. Evo will either be reflected from the barrier or will tunnel through the barrier. The total number of reflections plus tunnelings is always 50. You need to count the number of times that Evo tunnels the barrier. The quantum tunneling probability is then your count divided by 50. Your goal is to determine the relationship between tunneling probability and barrier width.

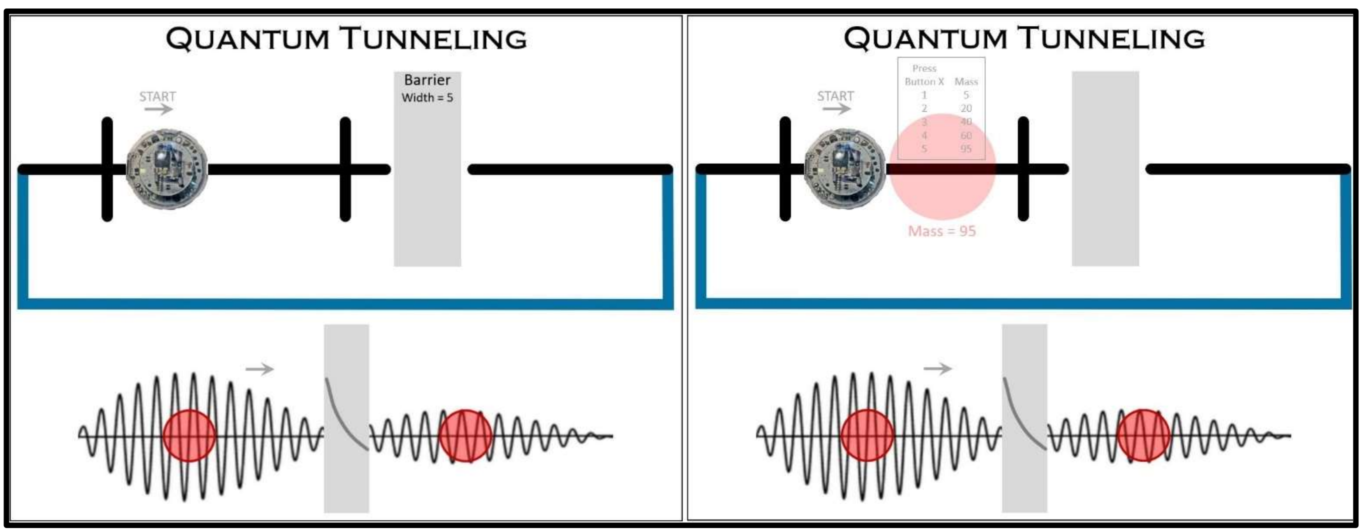

Figure 1

The right side of Figure 1 shows one of the Ozomaps that is used with the OzoBlockly program *Tunneling Mass.ozocode*. Evo is placed at the start location facing the direction of the arrow. The program is started. When the top light turns green, you have 8 seconds to press the button either 1, 2, 3, 4, or 5 times. The table on the Ozomap indicates how the number of button presses corresponds to the subatomic particle's mass. For example, five button presses simulate a particle mass of 95 units. Each time you press the button, the top light will quickly flash white to let you know that Evo recognized the button press. After the 8 seconds have elapsed, Evo will speak the value of the mass and begin moving. Evo will either be reflected from the barrier or will tunnel through the barrier. The total number of reflections plus tunnelings is always 50. You need to count the number of times that Evo tunnels the barrier. The quantum tunneling probability is then your count divided by 50. Your goal is to determine the relationship between tunneling probability and particle mass.

Your teacher will provide you with full-page copies of the Ozomaps. These can be used with Evo to perform this experiment.

Data Collection

Use the table of Figure 2 to record your data. Your teacher will ask you to enter your data into a spreadsheet with all of your classmates' data. The spreadsheet will show graphs of *tunneling probability versus barrier width*, and *tunneling probability versus particle mass*. Combining data from all classmates will smooth out the randomness of the results for a better analysis. The trendline feature of the spreadsheet will reveal the mathematical nature of the relationships between tunneling probability and the independent variables barrier width and particle mass.

Barrier Width (arbitrary units)	Tunneling Probability (%)		Particle Mass (arbitrary units)	Tunneling Probability (%)
1			5	
2			20	
3			40	
4			60	
5			95	

Figure 2

Questions

1. What do you conclude about the mathematical nature of the relationships between tunneling probability and the independent variables barrier width and particle mass?

2. Suppose that an electron and a proton with the same kinetic energy encounter identical potential barriers. Which one will tunnel through the barrier more easily? Why?

3. The probability that an object will tunnel through a barrier is given approximately by the equation

$$P = e^{-2KL},$$

where L is the width of the barrier and K is the wave number. The wave number is given by the equation

$$K = \frac{2\pi\sqrt{2m(V-E)}}{h},$$

where m is the mass of the object, V is the potential energy of the barrier, E is the kinetic energy of the tunneling object, and h is Plank's constant.

   Suppose that you have a mass of 60 kg, are moving at 1 m/s toward a barrier of width 1 m, and are trying to tunnel through a 40 Joule potential barrier. (A 40 Joule potential barrier is equivalent to tossing a 1 kg object 4 meters high, from the familiar equation $V = mgh$.) What is the probability that you tunnel through?

4. Consider an electron having a kinetic energy E of 4.75 eV. It is approaching a barrier with potential V of 5.00 eV and a width L of 1000 pm (picometer = 10^{-12} m). Since $E<V$, classical physics tells us that the probability is zero that it passes through the barrier. Calculate the quantum probability of the electron tunneling through the barrier.