

Game Tree for the Game of Nim:
An Exercise for Computer Science Students

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Figure 1 shows a portion of the game tree for Nim with the number of matchsticks $n = 6$. Each node of the tree represents a board configuration showing the number of matches remaining. The root node represents the starting configuration with six matches. Transitions from one level to the next are made alternately by player A (green) and player B (blue). Numbers on the branches indicate the number of matches removed by the player. Leaves of the tree are shown as yellow circles with the winning player indicated in the leaf.

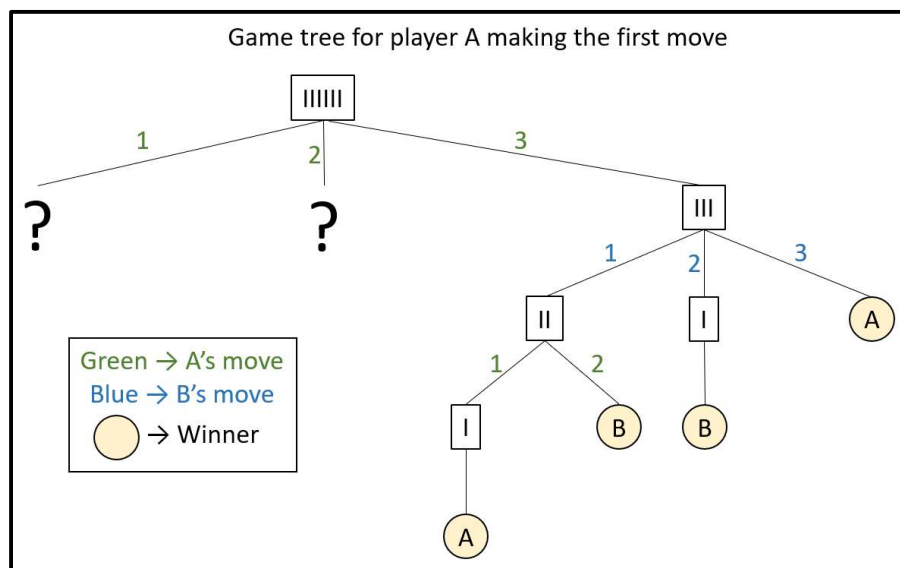


Figure 1

Figure 1 assumes that player A (green) will be making the first move. The complete tree for the case where player A elects to remove 3 matches from the initial 6 matches is shown in the figure. After A's initial move, there are 3 matches remaining. Player B (blue) can then elect to remove 1, 2, or 3 of the remaining matches:

- Suppose that B removes all three of the remaining matches. This would clearly be a mistake on B's part, for B would be removing the last match and lose. Player A would be the winner, as shown by the yellow circular leaf.

- Suppose that B removes 2 of the remaining 3 matches. That would leave one match remaining, and player A would have to remove it losing the game. Here, B is the winner, as shown by the yellow circular leaf.
- Suppose that B removes just 1 of the remaining 3 matches. That leaves 2 matches and it is A's turn again. A can remove either one or both of the remaining two matches:
 - Suppose that A removes both. This would be a mistake on A's part since by doing so, A would be removing the last match and lose. B would be the winner.
 - Suppose that A removes just one of the remaining 2 matches. This would leave one match, and B would have to remove it, making B the loser. A would be the winner.

Hopefully, you can see that game trees are useful in determining what a player's next move should be. Similar trees could be constructed for finite games such as tic-tac-toe.

Your Challenge as a Student of Computer Science

You will notice that there are two large question marks in the game tree of Figure 1. Your job is to complete the tree for each of these question marks. They correspond to player A's initial move of removing either 1 or 2 matchsticks. You will likely need an entire sheet of paper to complete the tree for either of these two moves. (As a check on your work—there will be 11 leaves if player A initially removes 1 matchstick, and 9 leaves if player A initially removes 2 matchsticks.)