Ozobot Classroom Application: Discovering the Golden Ratio

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Introduction

Due to its appearance with astonishing frequency in both mathematics and nature, the ***Golden Ratio*** has fascinated mankind for centuries. This ratio (approximately 1.618), symbolized by the Greek capital letter Φ (phi), *is a proportion which can be obtained by dividing a line segment into two pieces, in such a way that the ratio of whole segment to the longer piece is identical to the ratio of the longer piece to the shorter piece*. See Figure 1 for an illustration of this.

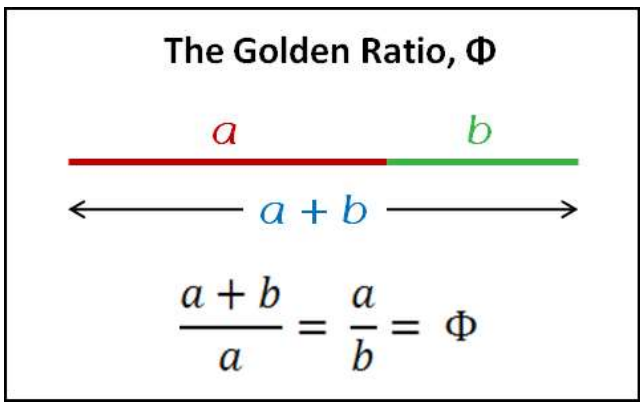


Figure 1

Whenever the lengths of the sides of a rectangle are in the golden ratio, then the rectangle is said to be a "golden rectangle." The rectangle surrounding Figure 1 is very close to being a golden rectangle. The golden rectangle is often considered to be aesthetically pleasing to the eye, providing great interest to artists and architects.

The Golden Ratio, CCSS, and Ozobot

Although you won't find "golden ratio" when searching the *Common Core State Standards Initiative* web site, you will find numerous standards in the *CCSS.MATH* area dealing with ***ratios*** and ***proportions***, especially in the standards for grades 6 through 8. The Golden Ratio provides a perfect stage for studying ratios and proportions, and using Ozobot to do so can be both fun and educational! In this classroom application, students will *discover*, via an experiment, the value of the Golden Ratio Φ by *timing* Ozobot as he travels along line segment *a+b*, and along pieces *a* and *b* (as shown in Figure 1). Students can use stop watches that read to nearest hundredth of a second, or they can use equivalent stop watch apps on their cell phones. Students will collect time data for four different line segments, one of which has pieces that are very near the Golden ratio. A data table is provided for students or student lab groups to record their timing data.

The OzoMap for this Classroom Application

For reference purposes while discussing how to interface Ozobot with this classroom application, Figure 2 shows a small image of the provided OzoMap. A larger version, intended for student use, can be found on the last page of this document and duplicated for student lab groups.

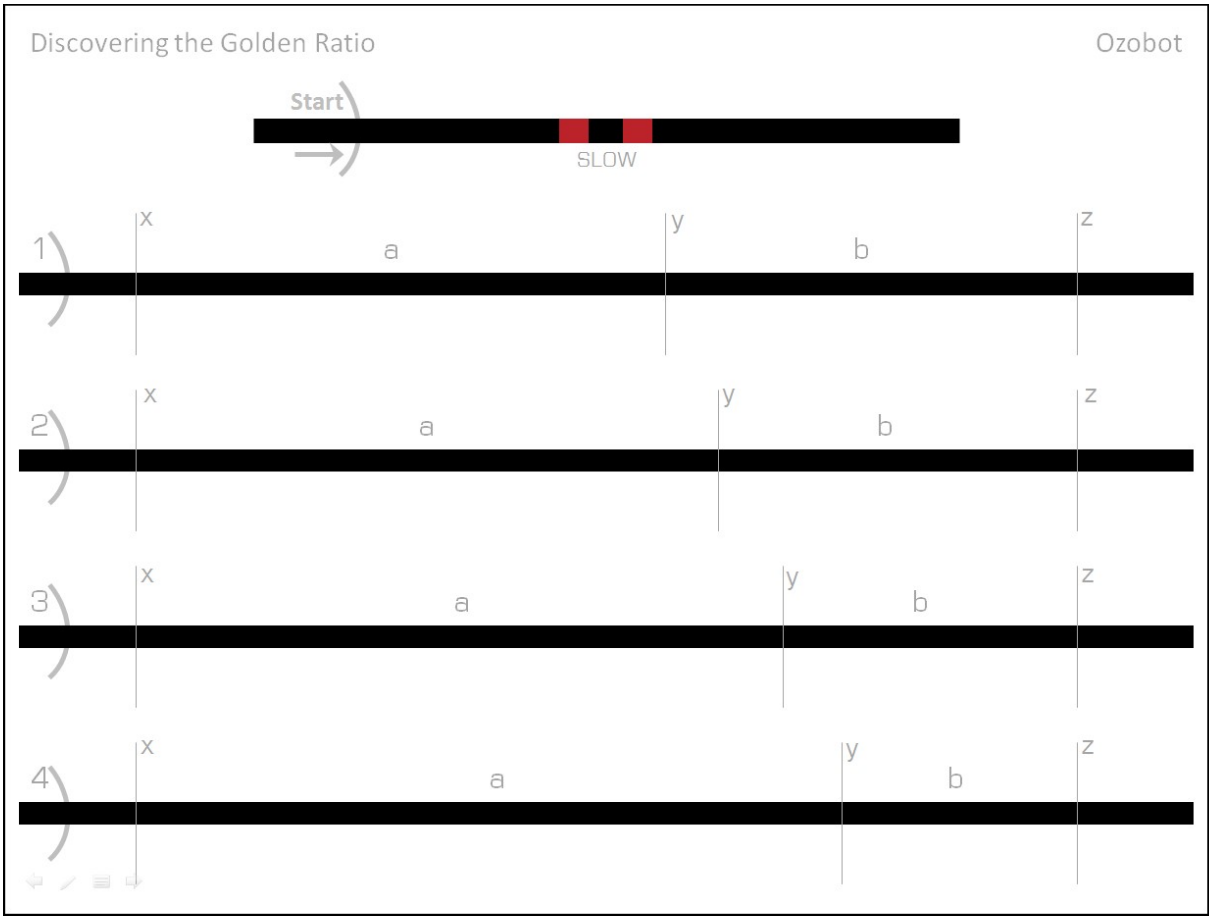


Figure 2

The start button on either Ozobot 1.0, Bit or Evo should be pressed once after placing him on the location labeled "Start." He will then encounter an OzoCode (Red-Black-Red) that tells him to move at the slowest speed allowed. The reason for this is to minimize the effect of reaction times when starting and stopping the stop watches. The slower that Ozobot is traveling, the smaller the ratio of reaction time to the time being measured, assuming that reaction time is independent of Ozobot's speed.

Ozobot can then be lifted off of the OzoMap and moved to the start position at the far left of each of the four lines, labeled 1 through 4. The short vertical gray lines labeled x, y, and z are provided as markers for starting and stopping stop watches. For each of the four lines, measurements should be made of:

- The time to traverse the entire segment from x to z, symbolized by t_{a+b}.
- The time to traverse piece *a* (from x to y), symbolized by t_a.
- The time to traverse piece *b* (from y to z), symbolized by t_b.

All times should be measured to the nearest hundredth of a second and recorded in the appropriately labeled columns of the data table on the next to the last page of this document. The students should finally compute the two ratios t_{a+b}/t_a and t_a/t_b, recording the values in the two rightmost columns of the data table. The students should find that one of the ratio pairs for the four lines will be much closer together than the other three, giving an approximation for the value of the golden ratio Φ! See Figure 3 for a table of typical results that lab groups may obtain.

While students could have obtained the value of Φ by using a ruler and measuring lengths instead of times, the design of Ozobot and its ability to maintain a constant speed makes timing the natural mode for measurements in this classroom application. This also provides the teacher with another point to bring up regarding proportions. Since *speed is constant* for Ozobot in this application and since distance equals velocity multiplied by time ($d = vt$), *distance is proportional to time,* with velocity being the constant of proportionality. This is precisely why we can use time measurements instead of distance measurements to determine the golden ratio.

For the Teacher: Typical Results
Discovering the Golden Ratio: Data Table

$$\frac{a+b}{a} = \frac{a}{b} = \Phi$$

Line Number	t_a (s)	t_b (s)	t_{a+b} (s)	t_{a+b}/t_a	t_a/t_b
1	5.24	4.12	9.55	1.82	1.27
2	5.88	3.58	9.51	1.62	1.64
3	6.51	2.90	9.42	1.45	2.24
4	7.13	2.43	9.39	1.32	2.93

Figure 3

Answers to Questions

1. Based upon the results in your data table, which one of the four line segments has pieces *a* and *b* that are nearest to the golden ratio? How can you tell*? [Line Number 2. The values in the two rightmost columns are most nearly the same.]*
2. What does your data table suggest as an approximation for the value of the golden ratio Φ? *[This will vary somewhat from one lab group to another, but should be reasonably close to 1.6.]*
3. Using a metric ruler, measure the lengths *a* and *b* of the line that you found to be closest to the golden ratio. Calculate the value of *a/b*. Does this value compare favorably with your answer to question 2? [a = 12.6, b = 7.8, Φ = 12.6 / 7.8 = 1.62. Agreement is very good.]

Preparation for the Lesson

1. Make a copy of the last two pages of this document for each student or lab group.
2. You can use either Ozobot 1.0, Ozobot Bit or Evo. ***No OzoBlockly programming is needed!***
3. The students should start their Ozobots by pressing the start button ***once***.

Discovering the Golden Ratio: Data Table

$$\frac{a+b}{a} = \frac{a}{b} = \Phi$$

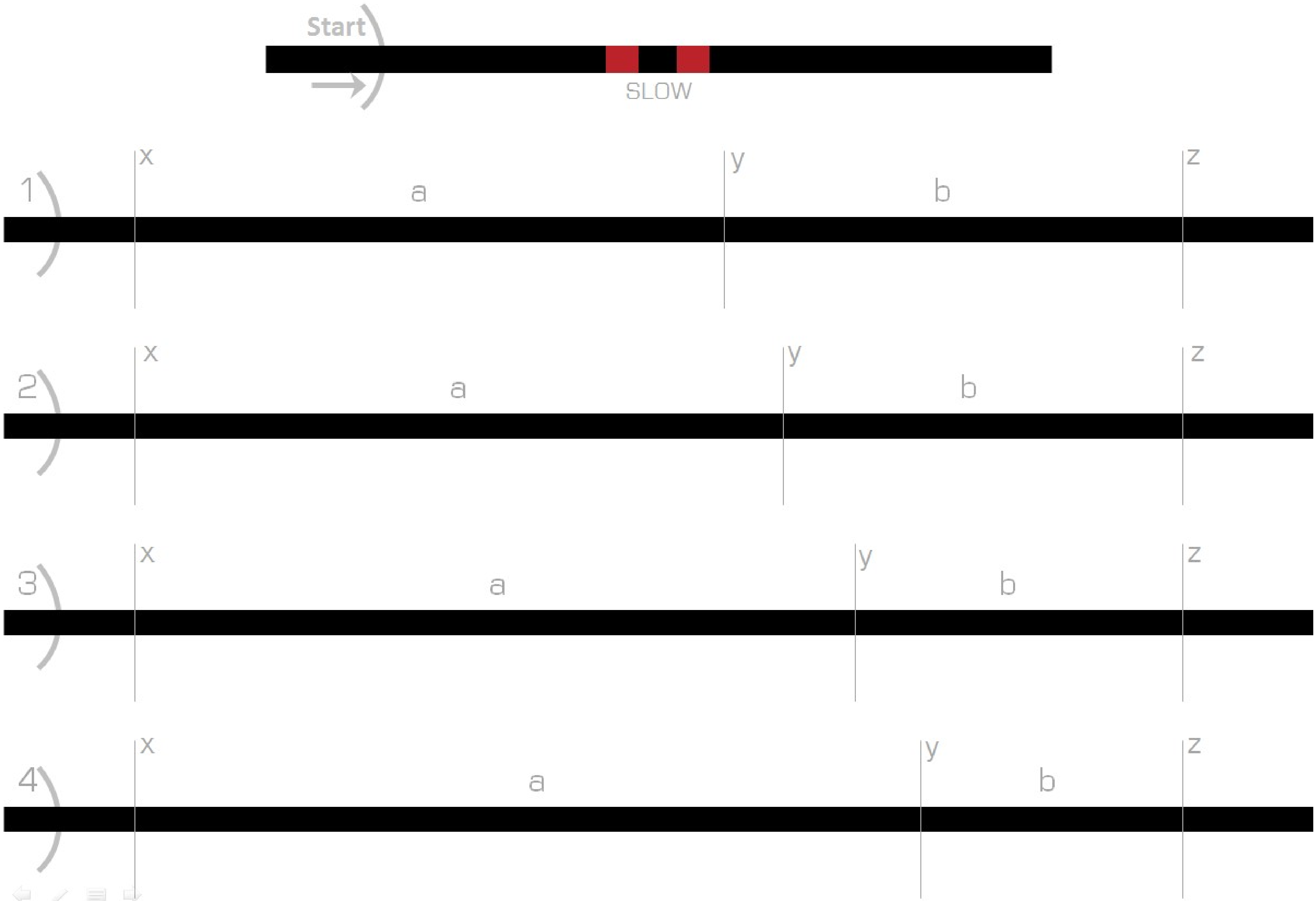
Line Number	t_a (s)	t_b (s)	t_{a+b} (s)	t_{a+b}/t_a	t_a/t_b
1					
2					
3					
4					

Questions

1. Based upon the results in your data table, which one of the four line segments has pieces *a* and *b* that are nearest to the golden ratio? How can you tell?
2. What does your data table suggest as an approximation for the value of the golden ratio Φ?
3. Using a metric ruler, measure the lengths *a* and *b* of the line that you found to be closest to the golden ratio. Calculate the value of *a/b*. Does this value compare favorably with your answer to question 2?

Additional Investigations (Optional)

1. (For algebra students) Using the defining equation at the top of this page, come up with a quadratic equation involving only Φ. Solve this equation for Φ using the quadratic formula.
    a. What is the value for the golden ratio Φ, to three decimal places, based upon the results from the quadratic formula?
    b. There is an extraneous root to this equation. Explain why it must be extraneous.
2. Research the WWW to find a way to construct a golden rectangle using only a straightedge and a compass. Try it out and measure the sides to see if the ratio of the longer to shorter side of the rectangle is golden.
3. Research the WWW to find out what the "Eye of God" refers to with regards to logarithmic spirals and what its relationship is to the golden ratio.
4. Research the WWW for applications of the golden ratio in nature.


Start
SLOW
1
x
a
y
b
z
2
x
a
y
b
z
3
x
a
y
b
z
4
x
a
y
b
z